New Formulas for the Trigonometry of Isosceles Triangles

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Abstract: This paper discusses a new formula to solve for the angle of isosceles triangles given proportion of differentiated unequivalent edge length versus the always-equivalent edges. The equation is equal to the arcsine of (X/2) multiplied by two. This formula is to be called 'isn^-1(x)', short for inverse isosceles sine equivalent. Also discussed is the non-inverse 'isn(x)' which is the cyclic function which is determined to correspond as the equivalent of the classic sine formula to isosceles triangles. The formula set is clearly superior and powerful at calculating the angle and measure of any given obtuse or acute triangle of an unclassified type, as well as unifying a simplest-fit formula across all types of triangle. Also discussed are two transform approximation formulas, which help to analyze graphs and samples. Finally, other methods your author has obtained to calculate angles are discussed and disproven in their effectiveness against this article's formulas at focus, as with any other known angle formulas.

Equations:

The inverse isosceles sine, with the inequal over one of the equal edge lengths as x:

$$isn^{-1}(x) = \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right) = \left(\frac{1*x^{(1)}}{(1)}\right) + \left(\frac{2*x^{(3)}}{(16*1*3)}\right) + \left(\frac{24*x^{(5)}}{(16\hat{}2*4*5)}\right) + \left(\frac{720*x^{(7)}}{(16\hat{}3*36*7)}\right) \dots$$

For the cyclic function, i= sqrt(-1) and with isn^-1 angle theta as x:

$$isn(x) = \sum_{n=1}^{\infty}\left(\frac{x^{2n}}{(2n)!} * (-1)^{(n+1)}\right)$$

$$\cong \left(\frac{x^2}{2} - \frac{x^4}{4*3*2*1} + \frac{x^6}{6*5*4*3*2*1} - \frac{x^8}{8*7*6*5*4*3*2*1} \ldots\right) \text{ and : } 1 - isn(x)$$

$$= \sum_{n=0}^{\infty}\left(\left(\frac{x^{2n}}{(2n)!}\right) * (-1)^{(n+1)}\right)$$

$$\cong \left(1 - \frac{x^2}{2} + \frac{x^4}{4*3*2*1} - \frac{x^6}{6*5*4*3*2*1} + \frac{x^8}{8*7*6*5*4*3*2*1} \ldots\right)$$

The equations appear to exponentiate upon the exact values of the angular and proportionate measures for the given notation, though may take large numbers of iterations to become precise.

In solving triangles besides isosceles, one can use isn^-1(x,y) which would be to parse x and y values of a point on the triangle in right triangles. The formula will be discussed with special reference to physics applications on the 'Additional Applications' section later. For using inverse isn isn^-1(a,b,c) to solve an unknown triangle with parts to where c is the opposite side length the angle, x in isn^-1(x) is defined:

$$x = 2 - \left|\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right|$$

All there is left to do in this scenario is multiply or divide the net angle of isn^-1(x) by two based on whether it is the smallest or largest side of the triangle.

Also, there is now 'Isosceles Inverse Cosine' ics^-1(x), to solve the angle of any triangle with three known side lengths, in a input-compressed fashion, or to solve such a triangle for 0-360 degrees instead of +/- 180 degrees:

$$ics^{-1}(x) = \left(\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right)\right) =$$

$$\pi - isn^{-1}(x) = (\pi - \sum_{n=0}^{\infty}\left(\frac{(2n)!\,x^{2n+1}}{16^n(n!)^2(2n+1)}\right) = \pi - \left(\frac{1 * x^{(1)}}{(1)}\right) +$$

$$\left(\frac{2 * x^{(3)}}{(16 * 1 * 3)}\right) + \left(\frac{24 * x^{(5)}}{(16^2 * 4 * 5)}\right) + \left(\frac{720 * x^{(7)}}{(16^3 * 36 * 7)}\right) \ldots$$

$$\ldots \textit{where in any triangle besides isosceles}:$$

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)}\right).$$

Additional Applications:

Another series of cyclic functions to be defined by modifications of isn(x) that will calculate and be equal to to proportion from the radius of the graph or angle to the respective x and y components on the cartesian plane is obviously managable from here. They are to be named isx(x) and isy(x).

$$isx(\gamma) = (1 - isn(\gamma))$$

$$isy(\gamma) = \left(1 - isn\left(\gamma - \left(\frac{\pi}{2}\right)\right)\right)$$

In physics, the isn^-1 equation is shown to be used to compute vector combinations and tensors more effectively. It can also be used in pure vector mathematics with ease.

Given right triangles as reference to x and y components plus magnitudes of forces, like so:

$$\textit{Given force or vector set } F(n\ forces) \textit{ and } P's(n\ components)$$

$$\textit{of any vector for example}:$$
$$Px = \{Px(n)\}$$
$$Py = \{Py(n)\}$$
$$Pf = \{Pf(n)\}$$

…the resultant force on a point or object will be given the components:

$$\gamma = \left(\left(\frac{Py}{\sqrt{Py^2}}\right) * ics^{-1}\left(\frac{2Px^2}{Px * Pf}\right)\right) or \ldots$$

$$\gamma = \left(isn^{-1} \left(\frac{Py\sqrt{(Pf - Px)^2 \ + (Py^2)}}{|Py| * Pf} \right) \right) \ and \ldots$$

$$Fx = \left(Pf * \left(isx(\gamma) \right) \right)$$

$$Fy = \left(Pf * \left(isy(\gamma) \right) \right) \ so\ that \ldots$$

$$The\ force's\ angle\ equals:$$

$$\theta = ics^{-1} \left(\frac{Fx^2 + \sqrt{Fx^2 + Fy^2}^2 - Fy^2}{Fx * \sqrt{Fx^2 + Fy^2}^2} \right)$$

$$\ldots and\ its\ magnitude\ equals:$$

$$\sqrt{Fx^2 + Fy^2}$$

This proves that the theorem is effective in calculating dimensional components of vectors in a vastly alternative and simplified method. Positive and negative x values are right at hand in this setup, while y values must be differentiated. This at least preserves the symmetry of the reference dimension and, using corresponding signs for if the angle to a radian is negative, one can obtain full 360 degree coordinates, or adapt the formula to more advanced purposes. Less formulas by far are needed for calculation of geometry and physics with this vector method, and the calculations can be put in one piece with less external operations or operators.

A calculation for the ideal circumference of the perfect circumscription of a known SSS triangle can be done as follows, where variable ‘side’ varies from the longest to shortest side of the triangle for the calculation of the correspondent circumscription, semiscription, or inscription:

$$x = \left(\frac{(adjacent\ 1)^2 + (adjacent\ 2)^2 - (opposite)^2}{(adjacent\ 1) * (adjacent\ 2)} \right)$$

$$C = \left((\pi + |isn^{-1}(x)|) \right) * side$$

Additionally, the ratio of a circle diameter to circumference PI = 3.14159… can be calculated using the infinite series simplified from isn^-1(x):

$$\pi = \sum_{n=0}^{\infty} \left(\frac{(2n)!\, 2^{-2n+1}}{(n!)^2 (2n+1)} \right)$$

One more theoretical set of expressions which works best using isn^-1(x) is the set which produces the average of all angles of x->y and x->z, as well as the one with the average of x->y and y->z, in a set of points within an object in equidistant and regular conditions. The average value of the sum pairs of these angles respectively condense unique irrational numbers that vary across a set of possible values based on the scale, rotation and perspective of an object or region in a graph. The single value produced by the set makes it possible to correspond a single number of many decimals to an expansion that reproduces the graph of the object, and in the latter case may be able to locate the object by its center of point sample density when it is part of a larger equation. This method relies only on the average of the sum of the defining angles to a region R at its perspective, the equation and process can be expressed as:

$$Graph(x,y,z) \ = \sum_{x,y,z=(x_1,y_1,z_1)}^{(x_m,y_m,z_m)} \left(\frac{(isn^\wedge - 1(x,y) + isn^\wedge - 1(z,x))^{\sqrt{x^2+y^2+z^2}}}{m} \right)$$

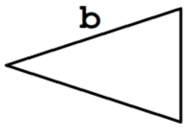
Where m will represent the number of x,y,z coordinates to fill the region that are used as a 3-dimensional list.

Simply put, this method will return irrational numbers that correspond uniquely and identically to one particular graph with optional transformations.

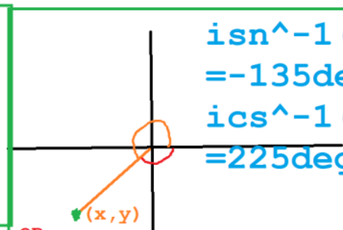
One other transform to reveal roughly the approximate harmonic signature of a series of samples represented as f(x) in terms of isn(x) frequencies(x) and amplitudes(y), and letting k represent the number of recorded samples of f(x):

$$Harmonics(f(x)) \ = \sum_{n=0}^{k} \left(\frac{isy(f(n) * x) * (\frac{k}{x+n})}{k} \right)$$

Diagram and Examples:



ang=
isn^-1(a/b)
OR
2 sin ^-1
(a/(2b))



isn^-1(x,y)
=-135deg
ics^-1(x,y)
=225deg

OR
tan^-1(x/y)
=positive/negative 45deg or undefined to a true quadrant

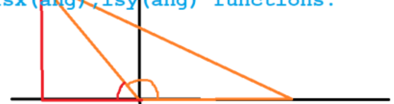
ISN	vs SIN
the isn(angle) is equal so that it is a scalar quantity that varies to twice the magnitude of vectors, as proportion so that it can derive possible reaction magnitudes when angles differ to a positive reference direction on things like normal forces	
isn converges faster for +/- 180 deg.	old trigonometry formulas do not converge as much, and slower
isn inverse can condense any points into a unique value	direction is indefinite using sin inverse
isn may be used to transfer force vectors across area	arcsin and sin and cos and tan aren't sufficient to define space as well
isn inverse unififes and simplifies law of cosines	every triangle needs a more complex adaptation with the old methods

LAW OF COSINES?

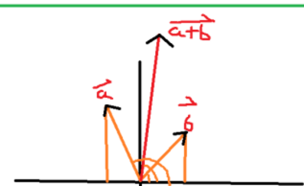
$$2*\hat{\sin}^{-1}\left(\left(2-\frac{b^2+c^2-a^2}{2bc}\right)/2\right) \text{ VS } isn^{-1}\left(2-\left(\frac{(adjacent\ 1)^2+(adjacent\ 2)^2-(opposite)^2}{(adjacent\ 1)*(adjacent\ 2)}\right)\right)$$

USE ISN^-1 (a,b,c) always

given angle or sides, isn^-1(a/b) OR isn^-1(x,y) = angle to +/- 180deg, which can translate pos/neg x and y thru isx(ang),isy(ang) functions.



given angle or sides, sin^-1 or tan^-1 or cos^-1 yields indefinitely orientied +/- 90 degrees of directional fidelity, and so sin(ang) and cos(ang) become indefinitely signed



using isn inverse on angles will give definitive angular measurements so that just an angle, can not be confused with an angle that is mirrored across positive x axis.

this means that sin inverse has been requiring us to ambiguously reduce our dimensional conceptualizations to indefinitively arranged vector components, and to sort them manually

PHYSICS (use as isn^-1(x,y))

SHORTCUT TO RIGHT TRIANGLE LAW OF COSINES IN ISN:

$$\left(\frac{Py}{\sqrt{Py^2}}\right) * ics^{-1}\left(\frac{2Px^2}{Px * Pf}\right) \quad \text{OR} \quad \gamma = \left(isn^{-1}\left(\frac{Py\sqrt{(Pf-Px)^2+(Py^2)}}{|Py| * Pf}\right)\right)$$

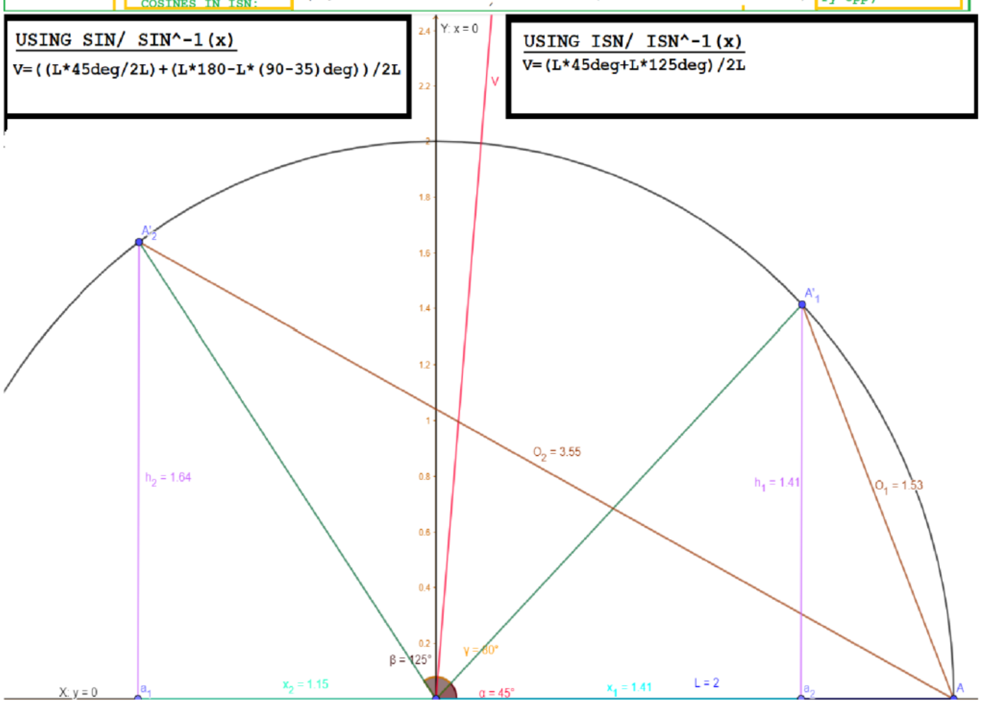
(Pf= hypotenuse
Px=adj
Py=opp)

USING SIN/ SIN^-1(x)

V=((L*45deg/2L)+(L*180-L*(90-35)deg))/2L

USING ISN/ ISN^-1(x)

V=(L*45deg+L*125deg)/2L



Discussion:

The isn, isn^-1 and ics^-1 formulas provide a unifying method whereat it is optimally easy to solve any other type of triangles (in relation to the relative component of the positions or sides of the adjacent points to the angle). The sin and arcsin methods are specific to components and do not provide method to solve other triangles in a way where the modifications are significant to accessible properties of the triangles.

Since no other formula has the proper blend of consistency, reflexive clarity and ease with calculating all types of triangles, and this formula is computationally superior for obtaining transformed physical predictions and simulations as well as data on geometric systems, ics^-1 ought to be used as a unifying trigonometric equation in the process of relating expressions and triangles to each other or their other measurements. it may very well be the superior method of calculating any type of triangle from an educational and computational perspective, as well as from a standpoint in search of algebraic abstraction of geometric properties.

Another method related to these equations as well as arcsin, whereat the 16^n at the denominator of the sum series expression may be substituted for 64^n. This results in a equation, that is equal to 4*arcsin(x/4) or 2*arcisn(x/2). The formula is so that f((+/-)4) is +/- (2*pi). One can theoretically abstract equations where adaptations of side lengths of any triangle be added in to solve for each, and the coordinate system can shift to handedness-biased +/- (2*pi radians, or 360 degrees). This requires extra factors, operations and calculations, and larger numbers to proportionate and converge on the answer. Conversely one can use the arcics function defined in this paper to handedness-relative convert a simulated rotation point to +/- 2*pi radians as well, and this is shown to work just as well if not better. For geometric and mathematical reasons, there is no reason to set a default measure so that the size of the number for an angle jumps randomly upon a full rotation. Simple left-and-right correspondence for half-circular rotations at a maximum of pi radians is much more effective for the person doing calculations. The major reason for this is because it is not relevant to as many forms of physical or theoretical geometry to suppose an ultimate direction or handedness of rotation for which an angle measure constantly increases in relation to any (or simply none at all of) references. Because angles are always relative, they should be so relative in any direction symmetrically to a reference direction. This is so that, for example, inverse isn formula can describe with definition a 3d angle from a reference origin and dimension, and any two chosen angles (between +/- pi) will always yield a single possible direction given the dimensions. The formula with 64^n at denominator would be yielding handedness-correlation issues which breaks the fundamental concept of graphical geometry and at least half of the possible +/- (2*pi) range directions made from two angles will be an ambiguous point, whereat the calculation could have been made in two different ways or represented by two different numbers. This breaks the uniqueness and mathematical value of otherwise unique graphs. Not only this, one final example is that, if the reference direction were changed to a different perspective on an objective with a reference point p and direction, some angle, such an angle would be entirely ambiguous to express in which way it has rotated if we are to assume a difference in rotation can imply not just a transition from the original angle. It would then be measured with respect to some fundamental and sometimes unknown coordinate, in the 64^n denominator case. Ultimately it will not be educationally better to use arcsin or the 64^n denominator case, nor technologically or in terms of efficiency or anything. This calls for the following ultimatum, that these isn, ics, etc. formulas are superior than any other possible way and should not be substituted or ignored.

Summary:

The expressional components of ‘ics^-1 ready’ triangular conversion coefficients for proper angle calculation are more universally supported by this formula than the cos^-1 theorem can rationally express to, making it computationally superior for closed systems and computing large networks of geometrically or mathematically bound variable systems that would correspond to everyday applied mathematics. Using a simple script and algebraic rules every geometric and physical formulation may be given a uniform distribution and ease of reference between different theorems, potentially simplifying together series of expressions in systems that are closed or of high complexity, not easily possible with sin or sin^-1 on an input or any scale of algebraically easily transformable level. It is suggested that many computational systems and graphing calculator make an easy use of this function or its generated dataset and its conversion utilities for a geometric, spatial or coordinate network.

Data Availability Statement, and Conflicts of Interest:

The data and calculations generated during and/or analyzed during this study were done by hand or graphed in GeoGebra with permission from the relevant copyright or license holders. Due to the fact that any standard calculator or convergence test will readily yield the results which are claimed in this text, they are readily available from the corresponding author on reasonable request.

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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